Brane Couplings from Bulk Loops*

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Abstract

We compute loop corrections to the effective action of a field theory on a fivedimensional S_1/Z_2 orbifold. We find that the quantum loop effects of interactions in the bulk produce infinite contributions that require renormalization by fourdimensional couplings on the orbifold fixed planes. Thus bulk couplings give rise to renormalization group running of brane couplings.

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1 Introduction

Recently it has been proposed that large extra dimensions may be relevant to particle physics at or near the weak scale [1]. This idea has opened up new possibilities for model building that make use of extra dimensions [2]. In [3], we studied a simple model of fermions and scalars interacting on a five-dimensional space with the fifth coordinate compactified on an S_1/Z_2 orbifold. In this model, the scalar field develops spatially varying vacuum expectation value resulting in a "fat brane" structure. The fermion field has a chiral zero mode that can be localized near either of the orbifold fixed points.

In this note, we continue our analysis of the model by computing loop corrections to the effective Lagrangian. The orbifold boundary conditions introduce two complications into the analysis. First, they break translation invariance (and hence momentum conservation) in the fifth dimension. Second, they single out two "fixed points" that are invariant under the Z_2 action on x_5 . As a result, couplings in the five-dimensional bulk can give rise to infinite effects that must be renormalized by couplings on the four-dimensional orbifold fixed planes. This renormalization is associated with running of the four-dimensional couplings on the fixed planes. In the following sections we develop the necessary formalism for computing perturbative corrections to the effective Lagrangian, and give examples of its use by computing the leading-logarithmic "brane terms" associated with renormalization group running for several special cases. A previous study of perturbative field theory on orbifolds can be found in [4]. This work considered a model with supersymmetric field theories living on the fixed planes, and discussed mechanisms for communicating supersymmetry breaking from one brane to the other.

In section 2, we write down the propagators on the orbifold. In sections 3 and 4, we discuss loop corrections. Section 5 contains conclusions and some ideas for further work.

2 Propagators

We consider a five dimensional Yukawa theory with the bulk action

$$\int d^5x \left\{ \overline{\psi} (i \partial - \gamma_5 \partial_5 - f \phi) \psi + (\partial \phi)^2 - V(\phi) \right\}. \tag{2.1}$$

The fifth dimension is compactified on a circle of circumference 2L with points on opposite sides of the circle identified. Thus, for instance, points $-x_5$ in $-L < -x_5 < 0$ are identified with points $+x_5$ in $0 < x_5 < L$. The points $x_5 = 0$ and $x_5 = L$ are invariant under the Z_2 action, and are referred to as fixed points. The fields are periodic with period 2L, and satisfy the boundary conditions

$$\psi(x, -x_5) = \gamma_5 \psi(x, x_5), \quad \psi(x, L + x_5) = \gamma_5 \psi(x, L - x_5), \tag{2.2}$$

and

$$\phi(x, -x_5) = -\phi(x, x_5), \quad \phi(x, L + x_5) = -\phi(x, L - x_5).$$
(2.3)

It was shown in [3] that this model possesses a single chiral fermion zero mode. In addition, for suitable $V(\phi)$, the scalar acquires a spatially varying vacuum expectation value (VEV) $\langle \phi(x_5) \rangle$. This spatially varying VEV can localize the chiral zero mode near either end of the orbifold.

Now consider the propagators in this model. If we ignore the boundary conditions, the fermion propagator is simply that of a massless five-dimensional fermion:

$$\frac{i}{p + i\gamma_5 p_5} \tag{2.4}$$

There are two differences on the orbifold. One is that there are true periodic boundary conditions when $x_5 \to x_5 + 2L$. This implies that

$$p_5 = \frac{\pi n}{L} \tag{2.5}$$

for integer n. The other difference is that because the physical region in the orbifold is smaller than the periodicity, momentum in the x_5 direction is not conserved. This is related to the reflection constraints at the orbifold boundary. An easy way to find the momentum space propagator is to notice that we can write ψ in terms of an unconstrained field χ as

$$\psi(x, x_5) = \frac{1}{2} \left(\chi(x, x_5) + \gamma_5 \chi(x, -x_5) \right). \tag{2.6}$$

This field automatically satisfies 2.2. We can now use this to compute the momentum space propagator. Notice that since both x_5 and $-x_5$ appear in (2.6), the propagator

$$S_5(x - x', x_5, x_5') = \left\langle \psi(x, x_5) \,\overline{\psi}(x', x_5') \right\rangle \tag{2.7}$$

depends on both $x_5 - x_5'$ and $x_5 + x_5'$. Doing the Fourier transform gives

$$\frac{i}{4} \left\{ \frac{\delta_{p_5 p_5'}}{\not p + i \gamma_5 p_5} + \gamma_5 \frac{\delta_{-p_5 p_5'}}{\not p - i \gamma_5 p_5} - \frac{\delta_{-p_5 p_5'}}{\not p + i \gamma_5 p_5} \gamma_5 - \gamma_5 \frac{\delta_{p_5 p_5'}}{\not p - i \gamma_5 p_5} \gamma_5 \right\}. \tag{2.8}$$

This can be simplified to

$$\frac{i}{2} \left\{ \frac{\delta_{p_5 p_5'}}{p' + i\gamma_5 p_5} - \frac{\delta_{-p_5 p_5'}}{p' + i\gamma_5 p_5} \gamma_5 \right\}. \tag{2.9}$$

Similarly, we can find the scalar propagator by rewriting ϕ in terms of an unconstrained field Φ as

$$\phi(x, x_5) = \frac{1}{2} \left(\Phi(x, x_5) - \Phi(x, -x_5) \right). \tag{2.10}$$

This gives a propagator

$$\frac{i}{2} \left\{ \frac{\delta_{p_5 p_5'} - \delta_{-p_5 p_5'}}{p^2 - p_5^2} \right\} . \tag{2.11}$$

3 Fermions

Now consider the one-loop correction to the fermion propagator from the diagram in fig.1. The fermion has momentum (p, p'_5) coming in and momentum (p, p_5) going out. Momentum is conserved at the vertices. So say that the incoming fermion splits into a fermion with momentum (k, k'_5) and a scalar with momentum $(p - k, p'_5 - k'_5)$. These propagate and the 5 components change drop their

primes and get reabsorbed. The internal loop momentum k is integrated and k_5 and k_5' are summed over. The diagram is then

$$i\Sigma = \frac{f^2}{4} \sum_{k_5, k_5'} \int \frac{d^D k}{(2\pi)^D} \left\{ \frac{\delta_{k_5 k_5'}}{\not k + i\gamma_5 k_5} - \frac{\delta_{-k_5 k_5'}}{\not k + i\gamma_5 k_5} \gamma_5 \right\} \left\{ \frac{\delta_{(p_5 - k_5), (p_5' - k_5')} - \delta_{-(p_5 - k_5), (p_5' - k_5')}}{(p - k)^2 - (p_5 - k_5)^2} \right\}.$$
(3.1)

Summing over k'_5 , the integrand becomes

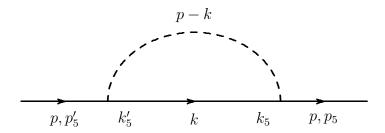


Figure 1: One loop correction to the fermion propagator.

$$\frac{1}{(\cancel{k}+i\gamma_5k_5)[(p-k)^2-(p_5-k_5)^2]}\left\{\delta_{p_5p_5'}+\delta_{p_5-p_5'}\gamma_5-\delta_{2k_5,(p_5+p_5')}-\delta_{2k_5,(p_5-p_5')}\gamma_5\right\} \tag{3.2}$$

$$=\frac{\not k+i\gamma_5k_5}{(k^2-k_5^2)[(p-k)^2-(p_5-k_5)^2]}\left\{\delta_{p_5p_5'}+\delta_{p_5-p_5'}\gamma_5-\delta_{2k_5,(p_5+p_5')}-\delta_{2k_5,(p_5-p_5')}\gamma_5\right\} \tag{3.3}$$

When we do the D-dimensional integral in (3.1), we encounter $1/\epsilon$ pole terms (where $D=4-2\epsilon$). In this paper, we consider only these divergent terms. For the pole terms, the p dependence comes only from the δ -functions and the numerator in (3.3). The first two terms in braces in (3.3) give contributions where $|p_5|$ is conserved. These terms are contributions to the five-dimensional bulk fermion kinetic energy. However, the last two terms have a different structure. They do not conserve $|p_5|$ and therefore cannot be associated with any term in the bulk Lagrangian. Rather, they yield a sum of terms where $p_5 \pm p_5'$ changes by an even multiple of π/L . These terms give contributions to the action that depend only on the values of the fields at the orbifold fixed points $x_5 = 0, L$, and thus they renormalize the couplings on the brane. We can understand this by considering a generic momentum space operator like

$$\sum_{p_5=p_5'+2\pi n/L} \overline{\psi}(p, p_5) \Gamma \psi(p, p_5'), \tag{3.4}$$

where Γ is some Dirac matrix. Transforming this to position space gives

$$(\delta(x_5) + \delta(L - x_5))\overline{\psi}(x, x_5)\Gamma\psi(x, x_5). \tag{3.5}$$

The constraint that p_5 changes by an even multiple of π/L means that we get δ -functions at $x_5 = 0, \pm L, \pm 2L, \ldots$ We have explictly written the δ -functions that are singular in the physical region $0 \le x_5 \le L$. If all multiples of π/L were summed over, we would of course get δ -functions at $x_5 = 0, \pm 2L, \pm 4L, \ldots$

Explicitly evaluating (3.1), we encounter a divergent piece

$$i\Sigma = \frac{-i}{4} \frac{f^2}{16\pi^2} \left[p \left(\frac{1+\gamma_5}{2} \right) + ip_5 \left(\frac{1+\gamma_5}{2} \right) - ip_5' \left(\frac{1-\gamma_5}{2} \right) \right] \frac{1}{\epsilon} + \dots$$
 (3.6)

When we eliminate the pole by minimal subtraction, we are renormalizing a brane term. This contributes to the running of the corresponding term on the brane. Subtracting and converting back to position space gives the contribution to the effective Lagrangian:

$$\delta L_{\text{eff}}(\mu) = \frac{1}{2} \frac{f^2}{16\pi^2} \log\left(\frac{\mu}{M}\right) \left[\delta(x_5) + \delta(x_5 - L)\right] \left[\overline{\psi}_+ i\partial \psi_+ + (\partial_5 \overline{\psi}_-)\psi_+ + \overline{\psi}_+(\partial_5 \psi_-)\right]$$
(3.7)

where $\psi_{\pm} = (1/2)(1 \pm \gamma_5)\psi$.

4 Scalars

In this section, we consider the divergent contributions to loops involving external scalars. The one-loop scalar tadpole is shown in Fig. 2. This diagram yields

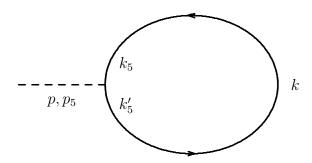


Figure 2: One loop contribution to the scalar tadpole.

$$\frac{f}{2} \sum_{k_5} \int \frac{d^D k}{(2\pi)^D} \operatorname{Tr} \frac{(\cancel{k} + i\gamma_5 k_5)(\delta_{k_5, k_5 + p_5} - \gamma_5 \delta_{k_5, -(k_5 + p_5)})}{k^2 - k_5^2}, \tag{4.1}$$

where we have used momentum conservation at the vertex to write $k'_5 = k_5 + p_5$. As before, the first Kronecker- δ has the form of a renormalization of the bulk Lagrangian (the coefficient vanishes in this case), while the second yields a brane term. Evaluating the integral with dimensional regularization and minimal subtraction gives

$$\delta L = \frac{f}{32\pi^2} \log\left(\frac{\mu}{M}\right) \left(\delta(x_5) + \delta(x_5 - L)\right) \partial_5^3 \phi. \tag{4.2}$$

In cutoff regularization, we would also find a quadratic divergence proportional to

$$(\delta(x_5) + \delta(L - x_5))\partial_5\phi. (4.3)$$

The effect of the $DR\overline{MS}$ term can be made more transparent by a change of variables in ϕ . For instance if the scalar potential vanishes, then we can eliminate the tadpole from the scalar sector of the theory by making the substitution

$$\phi = \phi' - \frac{f}{32\pi^2} \log\left(\frac{\mu}{M}\right) \left(\delta'(x_5) + \delta'(x_5 - L)\right). \tag{4.4}$$

This shift introduces a term proportional to $(\delta'(x_5) + \delta'(x_5 - L))$ fermion equation of motion.

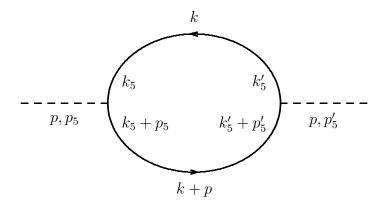


Figure 3: One loop correction to the scalar propagator.

The one loop contribution to the scalar propagator from the diagram in figure 3 gives no contribution to interactions on the brane. In this case, the loop integral is

$$-\frac{f^{2}}{4} \sum_{k_{5},k'_{5}} \int \frac{d^{D}k}{(2\pi)^{D}} \operatorname{Tr} \frac{(\cancel{k} + i\gamma_{5}k_{5})(\delta_{k_{5},k'_{5}} - \gamma_{5}\delta_{k_{5},-k'_{5}})}{k^{2} - k_{5}^{2}} \times \frac{(\cancel{k} + \cancel{p} + i\gamma_{5}[k'_{5} + p'_{5}])(\delta_{k_{5} + p_{5},k'_{5} + p'_{5}} - \gamma_{5}\delta_{k_{5} + p_{5},-k'_{5} - p'_{5}})}{(k + p)^{2} - (k'_{5} + p'_{5})^{2}}.$$

$$(4.5)$$

The brane terms vanish, since they are proportional to traces of odd numbers of γ matrices, or traces of fewer than four Dirac matrices with γ_5 . From symmetry considerations alone, one might have expected to find brane terms proportional to $(\partial_5 \phi)^2$. At higher loops, such terms are indeed generated. To investigate this, let's consider the two-loop graph in figure 4. Now consider the conservation of the 5 component of the loop momentum around the loop. Each of the propagators conserves the 5 component of the momentum it carries up to a factor of ± 1 . Call these factors η_5 , and associate the η_5 with propagators as shown in figure 5. Then we have

$$k_5 - p_5 = \eta_1 k_5'', \quad l_5 - p_5' = \eta_2 l_5'', \quad k_5 = \eta_3 k_5', \quad l_5 = \eta_4 l_5', \quad k_5'' - l_5'' = \eta_5 (k_5' - l_5').$$
 (4.6)

Eliminating k'_5 , l'_5 , k''_5 and l''_5 from (4.6) gives

$$\eta_1 p_5 - \eta_2 p_5' = (\eta_1 - \eta_3 \eta_5) k_5 + (\eta_2 - \eta_4 \eta_5) l_5.$$
(4.7)

We get brane terms when the right hand side of (4.7) does not vanish. It vanishes only when $\eta_1\eta_3\eta_5 = \eta_2\eta_4\eta_5 = 1$. This result is easy to remember. η_1 , η_3 and η_5 are the η_5 associated with the

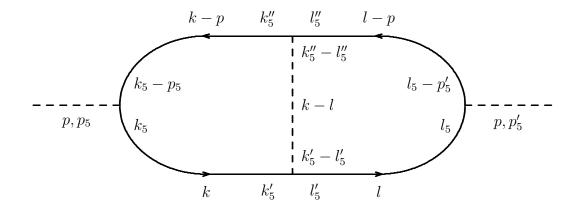


Figure 4: Two loop correction to the scalar propagator.

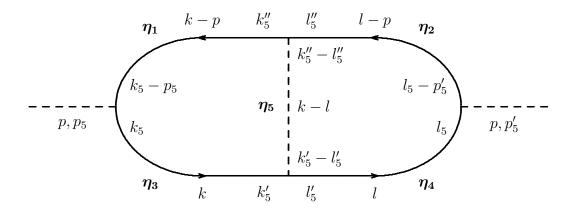


Figure 5: The association of the η s with the propagators in figure 4.

k loop and η_2 , η_4 and η_5 are the η_8 associated with the l loop. The product in each loop must be +1 to give a bulk term. Otherwise we get a brane term. This works for the one-loop diagrams as well, so we may speculate that it is a general result.

Of course, there is a second issue, namely whether or not the Dirac trace in the self-energy diagram vanishes. This gives a second constraint on the η 's. It is easy to see that we must have $\eta_1\eta_2\eta_3\eta_4=+1$ to get a non-zero result.

It is clear that from diagrams like figure 4 we can get brane contributions with an even number of ∂_5 s. Perhaps we should not read too much into their absence at the one-loop level.

Next consider the one-loop contribution to the scalar three point function in figure 6. The contribution to the effective Lagrangian is constrained by the boundary conditions. A ϕ^3 term is inconsistent with the boundary conditions, whether it is in the bulk or on the brane. Terms like $\phi^2 \partial_5 \phi$ are consistent with the boundary conditions, but vanish on the brane. The lowest dimension operator that is non-zero on the brane and consistent with the boundary conditions is $(\partial_5 \phi)^3$. Now

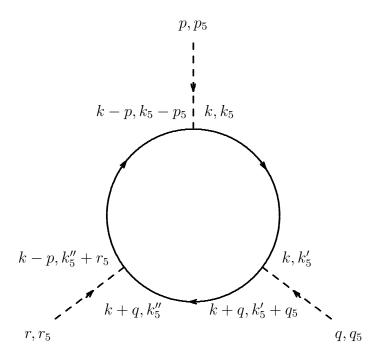


Figure 6: One loop contribution to the scalar 3-point function.

consider the loop integral. Labelling the momenta as shown in figure 6, the loop integral is

$$\frac{f^{3}}{8} \sum_{k_{5},k'_{5},k''_{5}} \int \frac{d^{D}k}{(2\pi)^{D}} \operatorname{Tr} \frac{(\cancel{k} + i\gamma_{5}k'_{5})(\delta_{k_{5},k'_{5}} - \gamma_{5}\delta_{k_{5},-k'_{5}})}{k^{2} - k'^{2}_{5}} \\
\times \frac{(\cancel{k} - \cancel{p} + i\gamma_{5}(k_{5} - p_{5}))(\delta_{k_{5} - p_{5},k''_{5} + r_{5}} - \gamma_{5}\delta_{k_{5} - p_{5},-k''_{5} - r_{5}})}{[k + q]^{2} - [k_{5} - p_{5}]^{2}} \\
\times \frac{(\cancel{k} + \cancel{q} + i\gamma_{5}k''_{5}))(\delta_{k''_{5},k'_{5} + q_{5}} - \gamma_{5}\delta_{k''_{5},-k'_{5} - q_{5}})}{[k - p]^{2} - k''^{2}} \tag{4.8}$$

As in the case of the self-energy diagram, the brane terms come from cross terms where the 5 component of the loop momentum undergoes an odd number of sign changes as it flows around the loop. Contributions to the running come from brane terms with two or three powers of the loop momentum in the numerator. We can see that there is no ϕ^3 term on the brane: this is simply because the portions of the integrand that would yield such a term are proportional to traces of the form

$$\operatorname{Tr} \gamma_{\mu} \gamma_{\nu} \gamma_{\lambda} \gamma_{5} = 0. \tag{4.9}$$

We can also see that no term of the form $\phi^2 \partial_5 \phi$ is induced. Such a term would vanish on the brane, but is nonetheless consistent with the boundary conditions. Collecting all terms in the numerator that are linear in the 5 components of the external momenta, we find a complete cancellation. This means that there are no terms of the form $\phi^2 \partial_5 \phi$, whether finite or infinite. We expect that the one-loop correction will, however, generate finite corrections with three or more derivatives.

Now consider the one loop correction to the $\phi \overline{\psi} \psi$ coupling shown in fig. 7. It's easy to see that the divergent part of the brane term vanishes. The divergence would come from terms with two

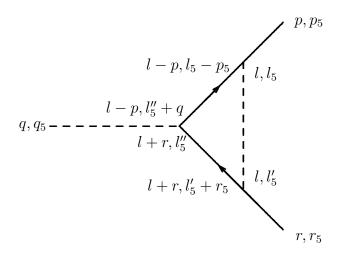


Figure 7: One loop correction to the Yukawa coupling.

powers of the loop momentum in the numerator of the integrand. A short computation shows that this piece of the numerator is proportional to

$$/// + ///\gamma_5 + //\gamma_5 // + //\gamma_5 //\gamma_5 = 0. (4.10)$$

Hence there are no infinite renormalizations of the $\phi\psi\overline{\psi}$ coupling on the brane.

5 Conclusions

Field theories on orbifolds may be a useful tool for model building in extra dimensions. We have shown that these theories necessarily have a hybrid structure, involving both five-dimensional bulk couplings and four-dimensional brane couplings. Under renormalization group flow, a theory with no brane couplings will generally flow to a theory with non-trivial physics on the brane. It is important to note that what we have discussed in this paper is the renormalization group running of couplings in the five-dimensional theory. Both the bulk and the brane couplings are defined in the five-dimensional theory, although by definition, the brane couplings appear in the Lagrangian with a δ -function that restricts them to the brane. This does not directly tell us about the running in the couplings in an effective four-dimensional theory derived from the five-dimensional physics, although it is surely a necessary component of any consistent calculation of this running. We hope to return to this issue and to study the particle physics implications of this result in future work.

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